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Hierarchy tests of edge transport models (BoRiS, UEDGE)

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Abstract

BoRiS is a 3D scrape-off layer (SOL) transport code under development to solve a system of plasma fluid equations. Using a simplified SOL model including particle continuity, parallel momentum and energy equations for both electrons and ions, BoRiS is tested in different geometries. To verify its proper operation in 1D and 2D cases, BoRiS solutions are compared to the results obtained with the established UEDGE code. In addition to these benchmarks some results for 3D problems are obtained.

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Keywords: SOL transport modelling; 3D modelling; Magnetic fusion edge plasmas; BoRiS code; UEDGE code; B2-Eirene code

1. Introduction

As a 3D SOL transport code solving a system of plasma fluid equations, BoRiS is strongly influenced by the experiences derived from modelling edge physics in fusion devices with model-validated 2D codes like B2-Eirene [1,2] and UEDGE [3,4]. BoRiS is being developed to describe edge physics phenomena in 3D systems like the new W7-X stellarator, as well as 3D effects in 2D configurations such as localized gas puffs in tokamaks or stellarators [5].

BoRiS is a general finite volume 3D code capable of dealing with mixed convection/conduction problems. It uses magnetic (Boozer) coordinates (s, θ, ϕ) to describe a complex 3D geometry. In this ansatz, standard discretization methods developed with other codes can be applied [6]. The physics model in BoRiS currently considers four equations for plasma density, parallel momentum and both electron and ion temperatures which represent a simplified edge physics model. Although simple, this model already reflects the main characteristics of an edge plasma, e.g., the competition between convection and diffusion depending on the plasma parameters. During its development process, our new code needs to be benchmarked with existing codes to prove the numerical technique and the reliabilty of the code [7]. Since there are many conceptional similarities, a direct comparison between BoRiS and UEDGE is of particular interest.

2. Set of equations

The benchmarks and tests described in this work were conducted with the following simplified SOL physics model:

$$\frac{\partial}{\partial t}n + \vec{\nabla}_{\|}(n\vec{u}_{\|}) - \vec{\nabla}_{\perp}(D_{\perp}\vec{\nabla}_{\perp}n) = 0, \quad (1)$$

$$\frac{\partial}{\partial t}(m_i n u_{\|}) + \vec{\nabla}_{\|}(m_i n u_{\|}^2 - \eta_{\|}\vec{\nabla}_{\|}\vec{u}_{\|}) + \vec{\nabla}_{\perp}(-m_i\vec{u}_{\|}D_{\perp}\vec{\nabla}_{\perp}n - \eta_{\perp}\vec{\nabla}_{\perp}\vec{u}_{\|}) = \vec{\nabla}_{\|}(p_e + p_i), \quad (2)$$

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$$\frac{\partial}{\partial t}\left(\frac{3}{2}nT_e\right) + \vec{\nabla}_\parallel\left(\frac{5}{2}T_e n\vec{u}_\parallel - \kappa^e_\parallel\vec{\nabla}_\parallel T_e\right) + \vec{\nabla}_\perp\left(-\frac{5}{2}T_e D_\perp\vec{\nabla}_\perp n - \kappa^e_\perp\vec{\nabla}_\perp T_e\right) = Q_{ei}, \quad (3)$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}nT_i + \frac{1}{2}m_i n u_\parallel^2\right) + \vec{\nabla}_\parallel\left(\frac{5}{2}T_i n\vec{u}_\parallel - \kappa^i_\parallel\vec{\nabla}_\parallel T_i\right) + \vec{\nabla}_\perp\left(-\frac{5}{2}T_i D_\perp\vec{\nabla}_\perp n - \kappa^i_\perp\vec{\nabla}_\perp T_i\right) = -Q_{ei}. \quad (4)$$

Here we have equal ion and electron density $n_i = n_e = n$, the parallel velocity $u_\parallel$ (defined according to $\vec{u} = \vec{u}_\parallel + \vec{u}_\perp = \vec{u}_\parallel - D_\perp\vec{\nabla}_\perp n/n$) and the temperatures T_e and T_i for both electrons and ions, respectively. The ion mass is m_i. In addition there are the parallel and perpendicular conductivities, $\kappa^a_\parallel$ and $\kappa^a_\perp$, the parallel and perpendicular viscosities $\eta_\parallel$ and $\eta_\perp$, and an anomalous diffusivity, $D_\perp$. The perpendicular conductivities and viscosities are also assigned anomalous values to model plasma turbulence. The source term Q_{ei} accounts for the heat exchange between electrons and ions, while $p_e = nT_e$ and $p_i = nT_i$ represent the thermal pressure of electrons and ions respectively. The above system (1)–(4) is written with respect to the physically relevant directions defined by the left-handed curvilinear system $\vec{B}$, $\vec{\nabla}s$, $\vec{\nabla}s \times \vec{B}$ ($\parallel$, $\perp_1$, $\perp_2$, respectively). The above time dependencies can be taken into account in order to resolve the time evolution of the system. A related development is a 3D fluid model for neutrals within the BoRiS framework [5]. An equation for the electrostatic potential will be included in the future.

3. 1D benchmark

The first test of BoRiS using the four plasma Eqs. (1)–(4) is for a simple 1D SOL physics problem (for hydrogen only). For this case all perpendicular ($\perp$) terms can be omitted from Eqs. (1)–(4). This 1D setup considers a magnetic field line connecting a point at the midplane with a point on the target plate. The length of this field line is chosen to be 35 m. The boundary conditions for the different quantities are as follows:

At the midplane the density is fixed to $n = 10^{19}$ m^{-3}, the parallel velocity is fixed to $0.01c_s$ ($c_s = ((T_e + T_i)/m_i)^{1/2}$ being the ion acoustic speed) and the temperatures of electrons and ions are fixed to $T_e = T_i = 100$ eV. At the target plate we assumed normal sheath conditions, which gives $u_\parallel = c_s$. Since there are no particle sources, the particle flux Γ_n is a constant, thereby determining the plate density to be $n = \Gamma_n/c_s$. The temperatures T_e and T_i are determined by their corresponding heat fluxes $\Gamma^{e,i}_q = \delta_{e,i}T_{e,i}\Gamma_n$ with the sheath coefficients $\delta_{e,i}$. The sheath coefficients were chosen to be $\delta_e = 5$ and $\delta_i = 3.5$. For our benchmark we consider a Coulomb logarithm of $\Lambda = 10$.

The results for this setup were compared to the ones obtained with two other edge transport codes (UEDGE and B2) solving the same problem. UEDGE uses the same numerical methods as BoRiS (see also the more detailed discussion in the last section), whereas B2 uses an iterative method solving the individual equations and then converging the complete set of equations by looping over them. These comparisons are shown in Figs. 1 and 2. While the electron temperature is almost constant, the ion temperature decreases slightly between the midplane and the target plate. According to this behaviour, the pressure gradient forcing the parallel velocity from $0.01c_s$ at the midplane to c_s at the target plate must almost completely come from the density which subsequently drops about two orders of magnitude. The small differences of the solutions of BoRiS, B2 and UEDGE for density and parallel velocity diminish with increasing grid resolution (see also discussion in the next section).

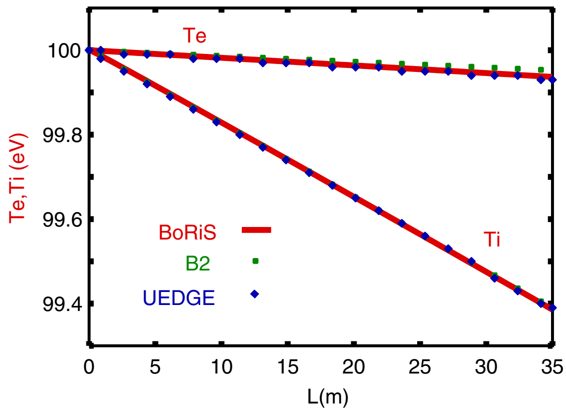

Fig. 1. Electron and ion temperatures along a field line.

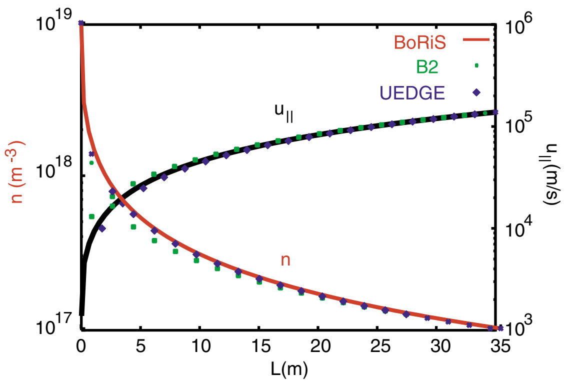

Fig. 2. Plasma density and parallel velocity along a field line.

4. 2D benchmark

A 2D test of BoRiS uses the basic setup of the 1D test case, but extended by adding another dimension ($\perp$) with appropriate boundary conditions. This configuration consists of a slab with overall dimensions characteristic for SOL physics phenomena: 35 m total length along the magnetic field, $\vec{B}$, and 0.1 m total width perpendicular to $\vec{B}$. Again the magnetic field lines connect one end which represents the midplane with another end representing a target plate. Along one side of the slab, we consider contact with a plasma and heat source prevailing over 70% of its entire length and thus representing a core boundary. The remaining 30% of this side is referred to as the private flux region. Accordingly the adjacent boundary represents the wall boundary of the plasma. The boundary conditions for this case are as follows:

At the core boundary,the density is fixed to $n = 10^{19}$ m^{-3}, the parallel velocity is fixed to $u_\parallel = 0$ m/s, and both the electron and ion temperatures are fixed to $T_e = T_i = 100$ eV. The midplane boundary is considered a symmetry plane and boundary conditions are set accordingly. For the parallel velocity this means $u_\parallel = 0$ m/s, while all other quantities have zero gradients. At the wall boundary, we assume all quantities to develop a gradient depending on an individual scale length l specifying an outflow condition. Here we choose $l = 5 \times 10^{-2}$ m for all quantities. At the private flux boundary, a condition similar to the one set at the wall is imposed, the only difference being steeper gradients resulting from $l = 1 \times 10^{-2}$ m for all quantities. At the target plate all quantities are again constrained sheath conditions. We assume $\Lambda = 10$ and anomalous perpendicular transport coefficients $\kappa_\perp^e = \kappa_\perp^i = \eta_\perp/m_i = D_\perp n$ with $D_\perp = 1$ m^2/s.

The results of our test are again compared to the results obtained with UEDGE solving the same problem. Figs. 3 and 4 show the 2D character of the solution and the quality of agreement between the two codes. The electron temperature is not shown since it varies mostly in the perpendicular direction (as T_i does), and the agreement between the two codes is again very close.

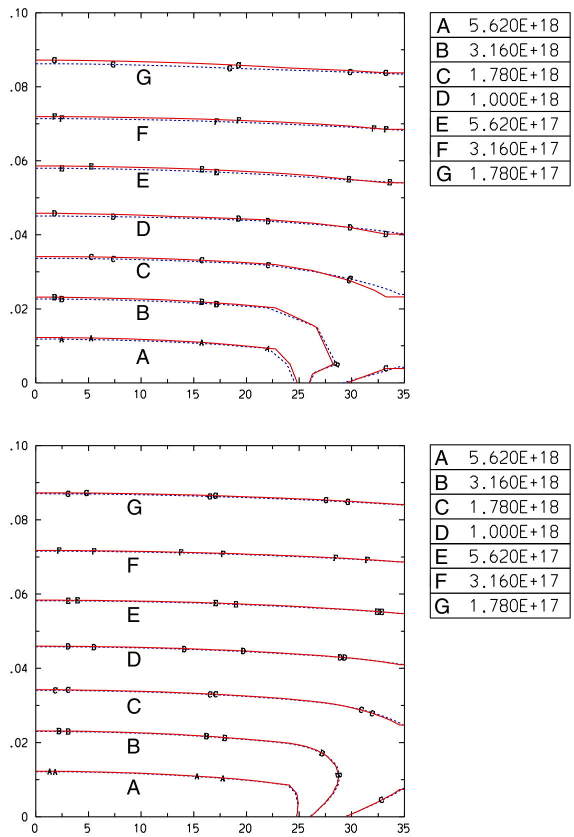

Fig. 4. Plasma density with a 10×10 (top) and 40×40 (bottom) mesh as obtained with BoRiS (solid red) and UEDGE (dotted blue).

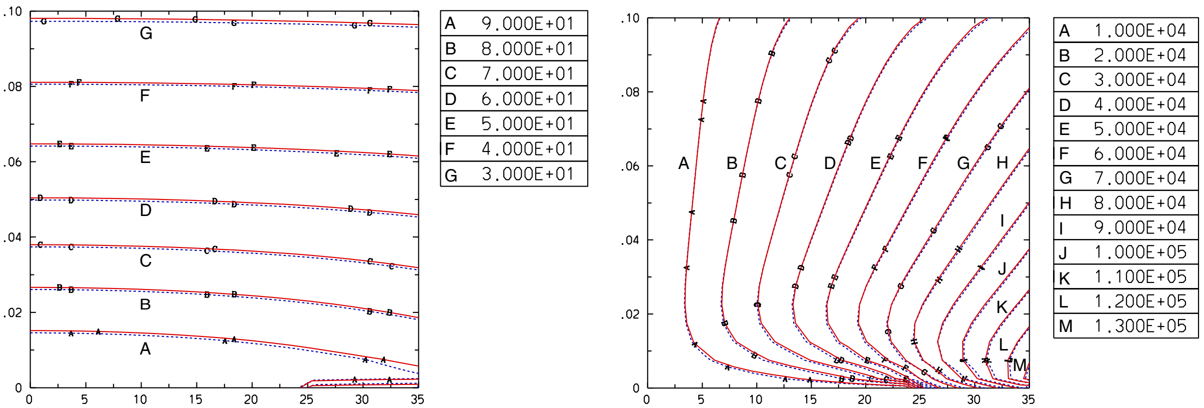

Fig. 3. Ion temperature (left) and parallel velocity (right) as obtained with BoRiS (solid red) and UEDGE (dotted blue).

According to the boundary conditions, the solution shows the characteristic features of this 2D problem. The core region feeds the domain with hot plasma by an influx due to the perpendicular gradient. The plasma is then accelerated along the field lines towards the target plate where it arrives with local ion acoustic speed. Along the private flux boundary there is an outflux of plasma being represented by a rather steep gradient in density and temperature. The maximum value of the parallel velocity is related to a pressure gradient which is almost exclusively caused by a significant drop of the density from upstream along the magnetic field to downstream, which is again due to the almost unchanged temperatures. The comparisons shown in Fig. 3 were obtained with a 20×20 mesh and are found to be in good agreement. However, there are slight differences found. The plots in Fig. 4 are to illustrate the origin of these differences. They show two results obtained with different mesh resolution. The upper plot corresponds to a 10×10 mesh and the lower one to a 40×40 mesh. The improved agreement for the 40×40 case indicates that the differences observed so far are only due to the differences in the meshes (see Section 6) used internally by the two codes.

5. 3D tests

In this section, we describe a 3D test case in a simple slab geometry with dimensions (x, y, z). To match the typical lengths in SOL physics, we choose $L_x = 0.1$ m, $L_y = 0.1$ m and $L_z = 35$ m. In BoRiS, this slab in real space geometry is internally represented by a corresponding slab in generalized coordinates (magnetic coordinates) (s, θ, ϕ) which are normalized to unity. In our case we choose (x, y, z) to be perfectly aligned with the direction of (s, θ, ϕ) respectively. We assume the following boundary conditions:

Both the $\phi = 0$ and $\phi = 1$ boundaries represent target plates with the appropriate sheath conditions being set for all quantities. At $s = 0$ we define a core boundary for $0.3 \leqslant \phi < 0.7$. Here, the density is again fixed to $n = 10^{19}$ m^{-3}, the parallel velocity is set $u_\parallel = 0$ m/s and both electron and ion temperatures are fixed to $T_e = T_i = 100$ eV. Also at $s = 0$, we define a private flux boundary with a unique scale length $l = 10^{-2}$ m constraining all quantities for both $0 \leqslant \phi < 0.3$ and $0.7 \leqslant \phi < 1$. The $s = 1$ surface represents a wall boundary and a unique scale length $l = 5 \times 10^{-2}$ m is set for all quantities. The $\theta = 1$ boundary is then connected with the $\theta = 0$ boundary periodically. To generate a 3D variation, we divided the $s = 0$ boundary at $\theta = 0.5$. For $0 \leqslant \theta < 0.5$ the conditions described above are maintained. For $0.5 \leqslant \theta < 1$ the boundary conditions are changed by setting a scale length $l = 10^{-2}$ m for all quantities. The results for this test as performed on a $10 \times 10 \times 20$ mesh are given in Figs. 5–7. As in the 2D case discussed above, the core boundary is characterized by its high density and high temperatures according to the boundary conditions. There is particle flow into the volume along the gradient perpendicular to the field and then along the field lines with increasing parallel velocity. Both the electron and ion temperature remain almost constant along the magnetic field and show a decrease perpendicular to it. Unlike our 2D case, this setup shows a symmetry around $\phi = 0.5$, which is most clearly visible in Fig. 6 showing the absolute value of $u_\parallel$. The 3D nature can be seen from the change of profiles taken at different θ positions as well as the periodicity in θ.

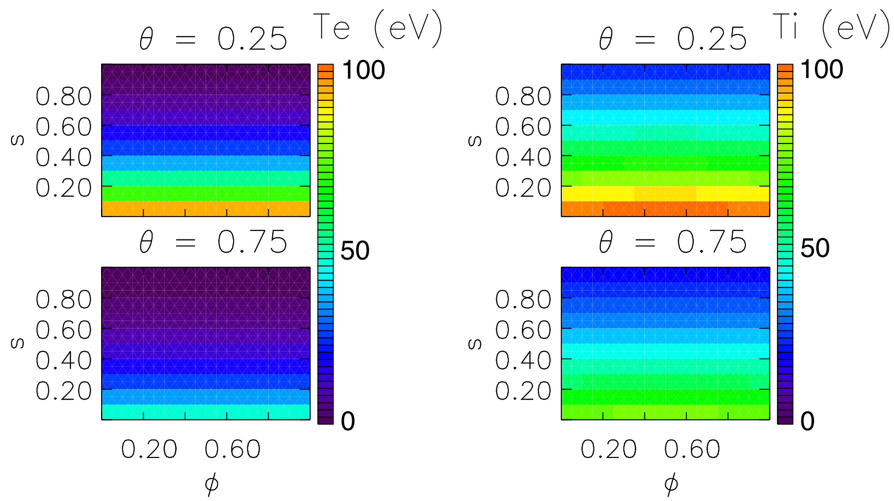

Fig. 5. Electron (left) and ion temperature (right) in a 3D slab at $\theta =$ constant.

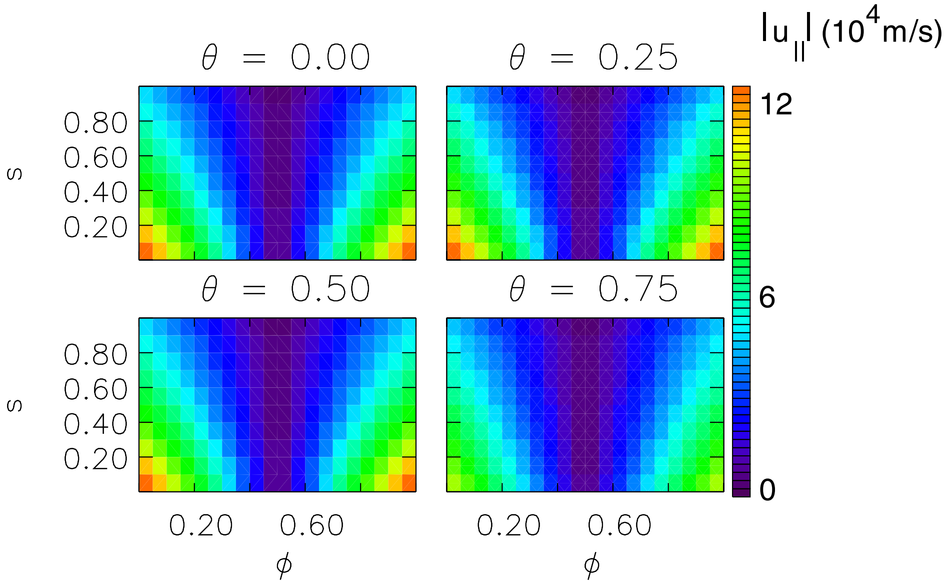

Fig. 6. Absolute parallel velocity in a 3D slab at $\theta = $ constant.

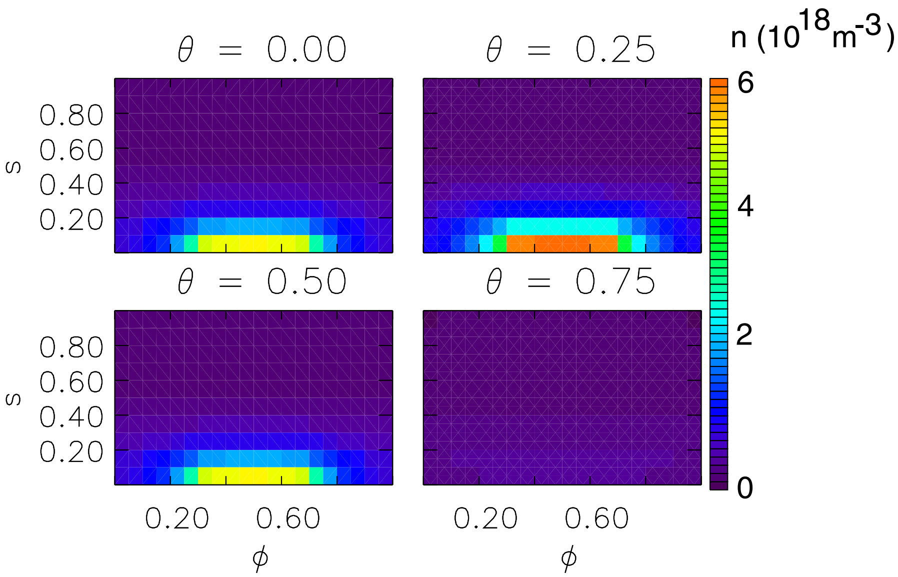

Fig. 7. Plasma density in a 3D slab at $\theta = $ constant.

6. Numerical performance

One reason to compare BoRiS and UEDGE on certain test cases is that they are similar in how to deal with a system of model equations being coupled. Both BoRiS and UEDGE solve for all quantities simultaneously using the Newton method. Moreover they utilize a variety of similar sophisticated solvers [6]. The results discussed in this work were obtained with a sparse iterative solver *GMRES(m)* in combination with ILU(0) and ILUT(p, τ) pre-conditioning in BoRiS and UEDGE respectively. According to these similarities, we compared the convergence behaviour of both BoRiS and UEDGE for two different variants of the above 2D benchmark. The result is given in Fig. 8. The first two curves in Fig. 8 correspond to a case where the calculation started from flat profiles for all quantities ($n = 10^{19}$ m^{-3}, $u_{\parallel} = 0$ m/s, $T_e = T_i = 100$ eV) with the boundary conditions described in Section 4. This calculation was performed in a real time dependent mode

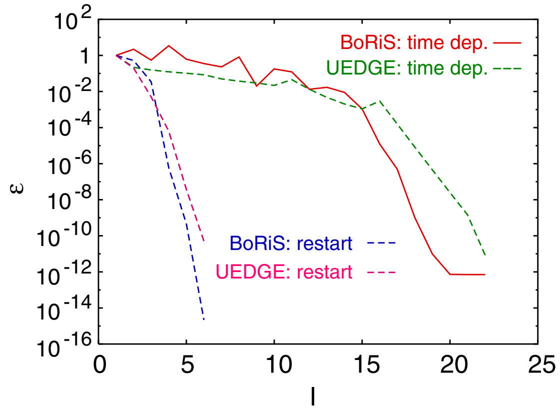

Fig. 8. Normalized residuals ϵ versus iteration number I.

with progressively increasing time steps and reaches convergence after 20 time steps. Both curves for BoRiS and UEDGE show a very similar behaviour even though there are some differences. The second pair of much faster decaying residuals corresponds to a case with infinite time step using the result of the first test as an initial guess, but changing the boundary conditions at the core to $n = 2 \times 10^{-19}$ m^{-3} and $T_e = T_i = 50$ eV. Again both codes show a very similar convergence behaviour. The differences observed so far may arise from the fact that the residuals considered were not exactly the same quantities and thus reflect the qualitative behaviour only.

As already mentioned in Section 4 there are some differences in the grids used internally by both BoRiS and UEDGE. These differences are in the exact locations of grid points and the positioning of guard cells which are used to impose boundary conditions. Another difference is the fact that UEDGE utilizes a staggered grid for the velocity while BoRiS has one grid for all quantities only.

In performing the tests described above we try to exclude all possible differences. For the interpolation this means that both BoRiS and UEDGE use 3-point and 5-point stencils for the 1D and 2D cases respectively. Along the parallel direction we allow for convection–diffusion corrections and use only linear interpolation in the perpendicular direction. In its 3D test case, BoRiS uses a 7-point stencil for interpolation.

7. Conclusions

A simplified SOL physics model including equations for the density, parallel momentum and both electron and ion temperatures is successfully introduced into BoRiS. With respect to its inherent similarities, BoRiS is tested on 1D and 2D problems and compared to the SOL transport code UEDGE. These comparisons show both the solutions and the numerical behaviour to be in agreement with the predictions derived from the UEDGE code. To show its capability of solving 3D problems, BoRiS is also tested on a 3D case, thus providing a hierarchy of test problems. Current work on BoRiS is focussed on the extension of the physics model, an extension towards more complex grids and on parallelization to meet the numerical needs of growing complexity.

References

[1] R. Schneider et al., Contribution Plasma Phys. 40 (3–4) (2000) 328.
[2] V. Rozhansky et al., Contribution Plasma Phys. 40 (3–4) (2000) 423.
[3] T.D. Rognlien et al., J. Nucl. Mater. 196–198 (1992) 347.
[4] T.D. Rognlien et al., Contribution Plasma Phys. 34 (1994) 362.
[5] M. Umansky et al., these Proceedings. PII: S0022-3115(02)01373-9.
[6] M. Borchardt et al., J. Nucl. Mater. 290–293 (2001) 546.
[7] A. Runov et al., Contribution Plasma Phys. 42 (2002) 169.